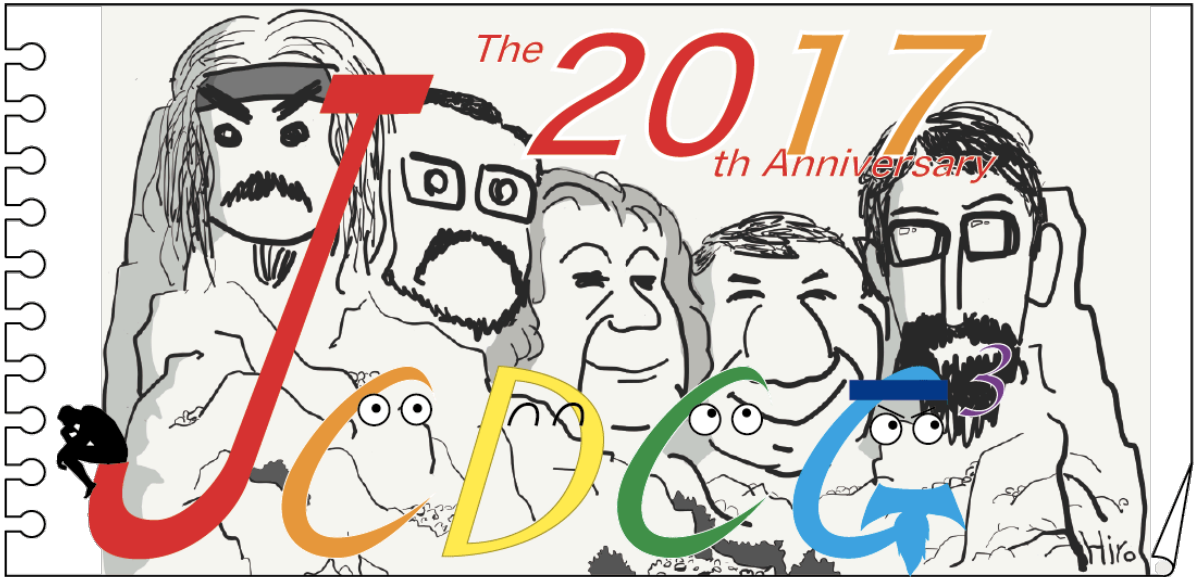

Planarity Preserving Augmentation of Plane Graphs to Meet Parity Constraints

J.C. Catana-Salazar, A. García, J. Tejel and J. Urrutia

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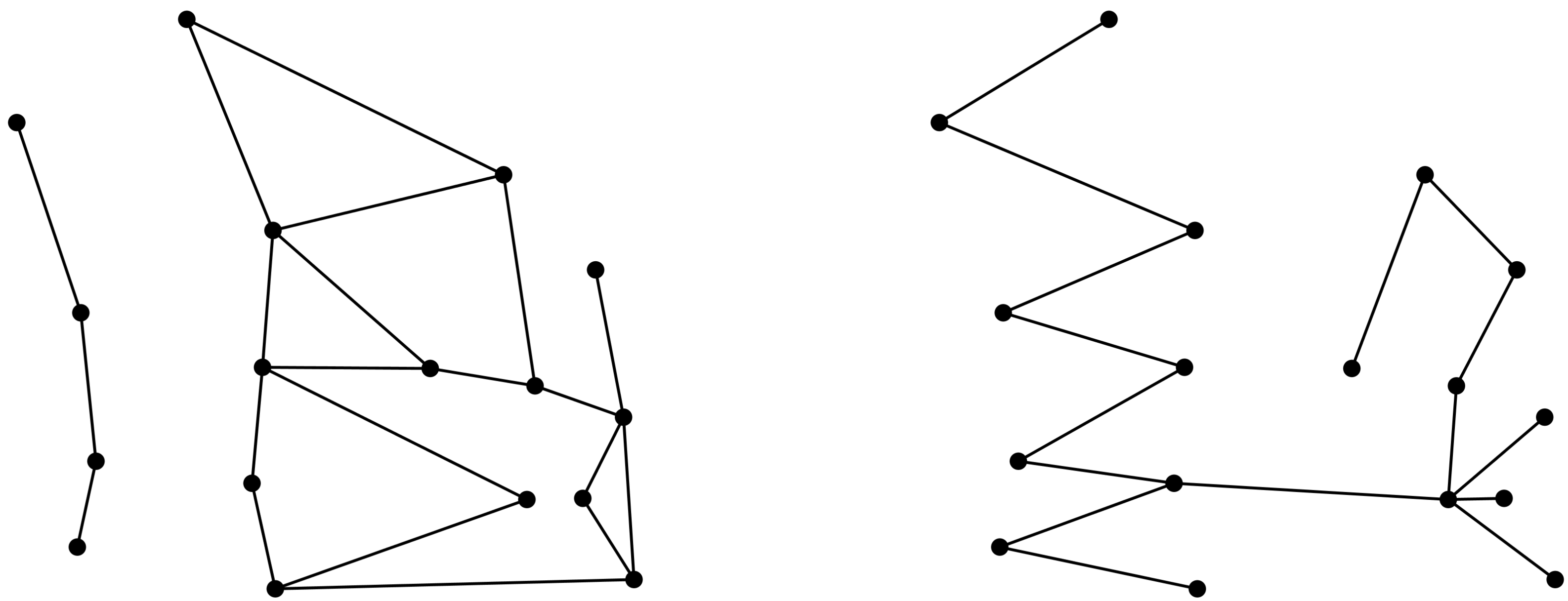
Definitions

- ***Compatible graphs***

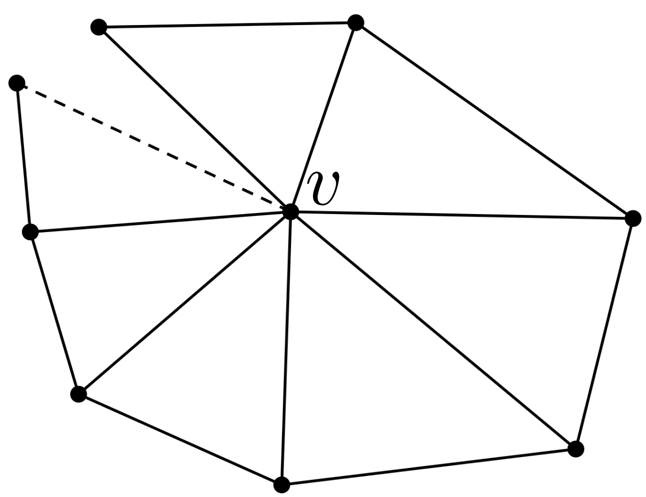
Two or more geometric graphs are compatible if their union is a simple plane graph



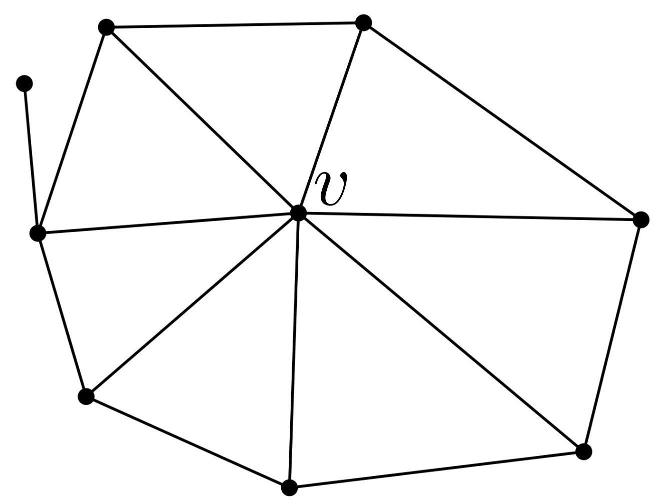
Definitions

- ***Saturated***

  The neighborhood of a vertex is saturated if there is no edge that can be added in the graph, incident to it, avoiding edge crossing.



v is not saturated



v is saturated

The problem

Given a topological (geometric) plane graph $G = (V, E)$ and a set of parity constraints $C = \{c_1, c_2, \ldots, c_n\}$ where each $v_i \in V$ has assigned the constraint c_i, the augmentation problem to meet parity constraints is that finding a set of edges E', where $E' \cap E = \emptyset$

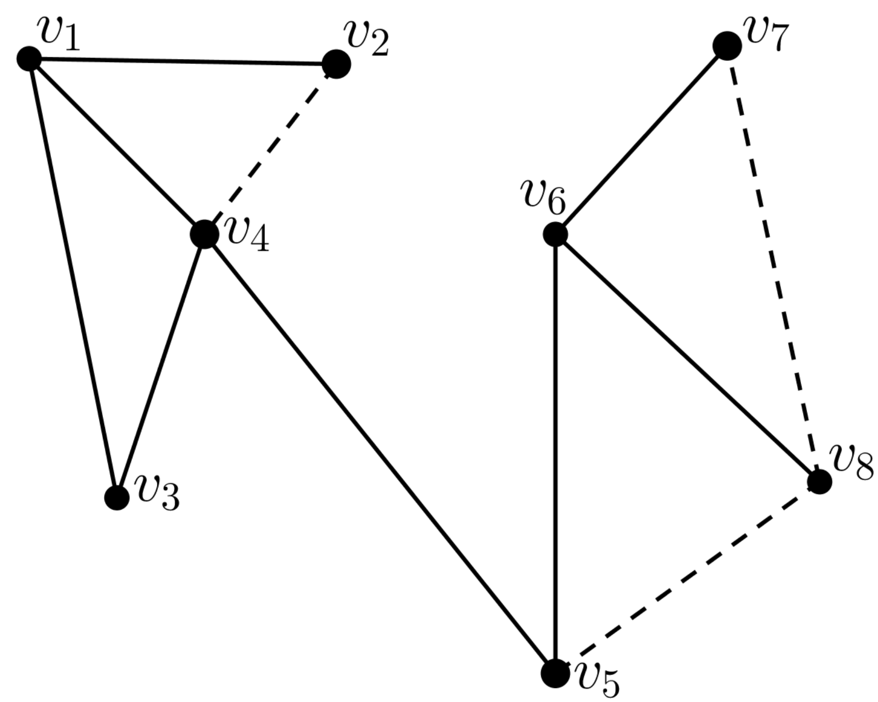
The problem

$$G = \{V, E\}$$

$$V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$$

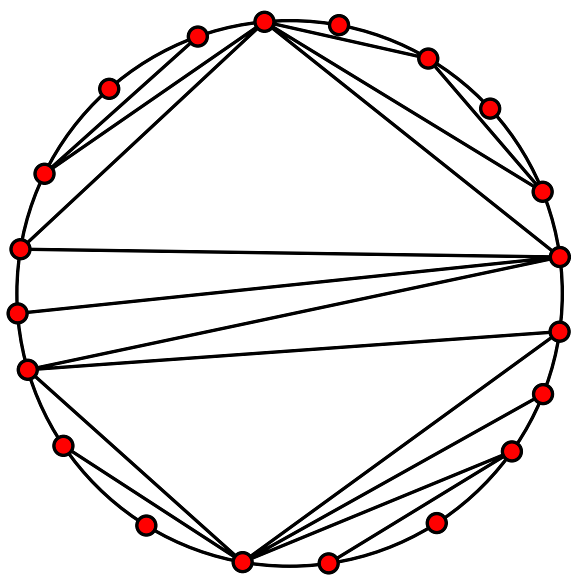
$$C_G = \{o_1, e_2, e_3, e_4, o_5, o_6, e_7, o_8\}$$

$$S(G, C_G) = v_2, v_4, v_5, v_7$$

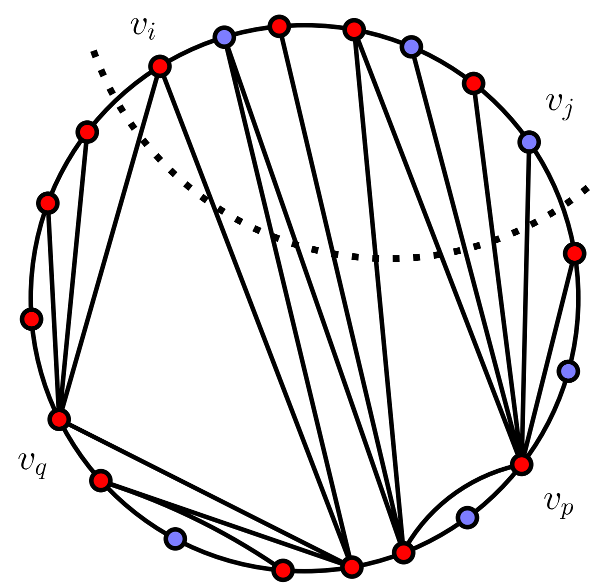


Fixed-embedding augmentation of MOPs

Non-augmentable graph family

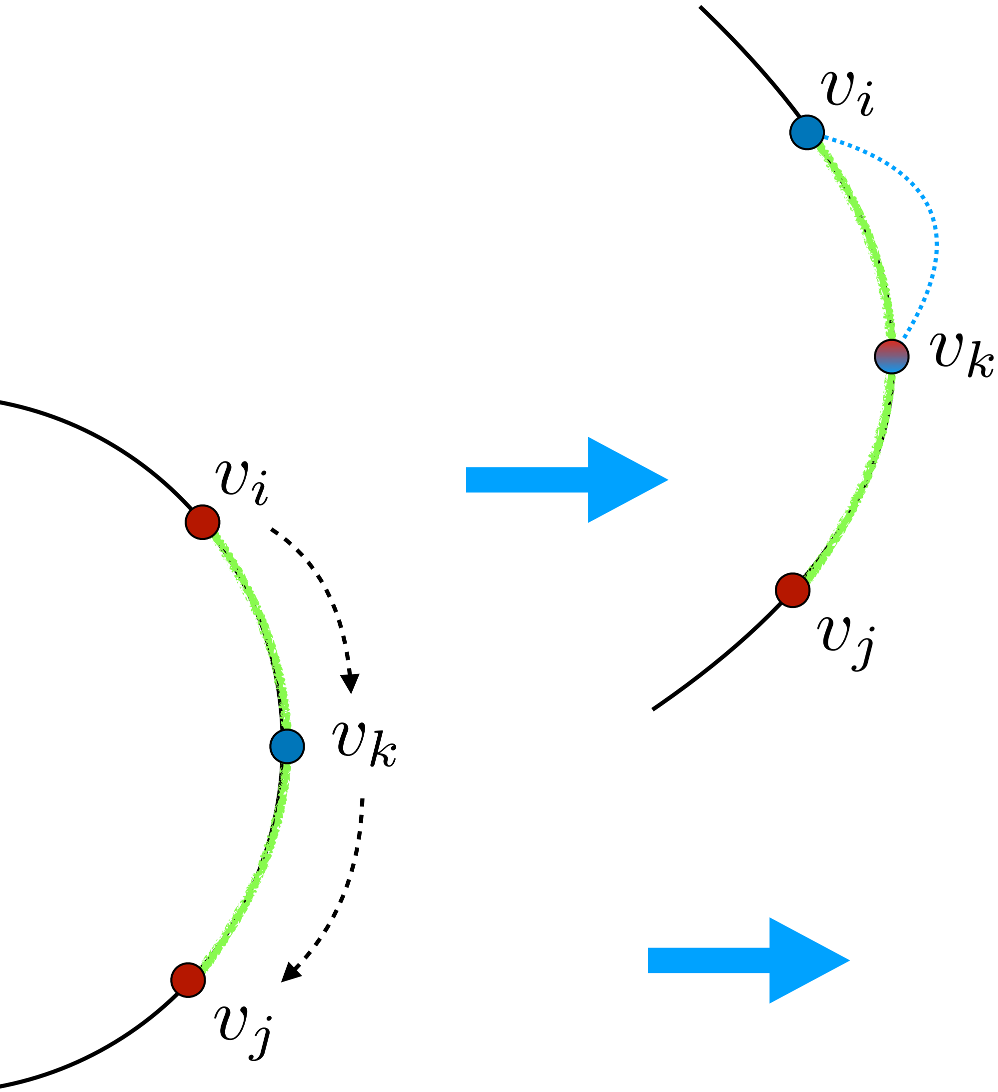


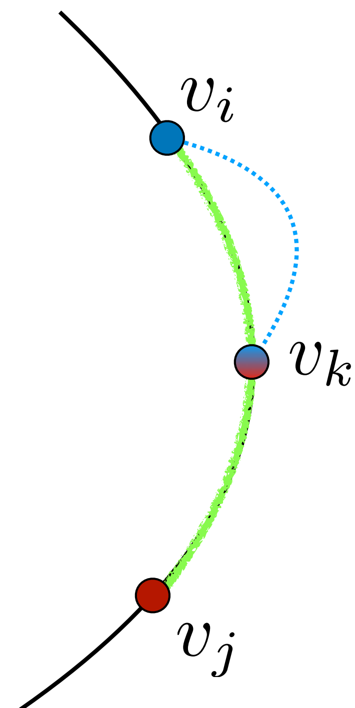
MOP with only red diagonals



MOP with only r-b diagonals

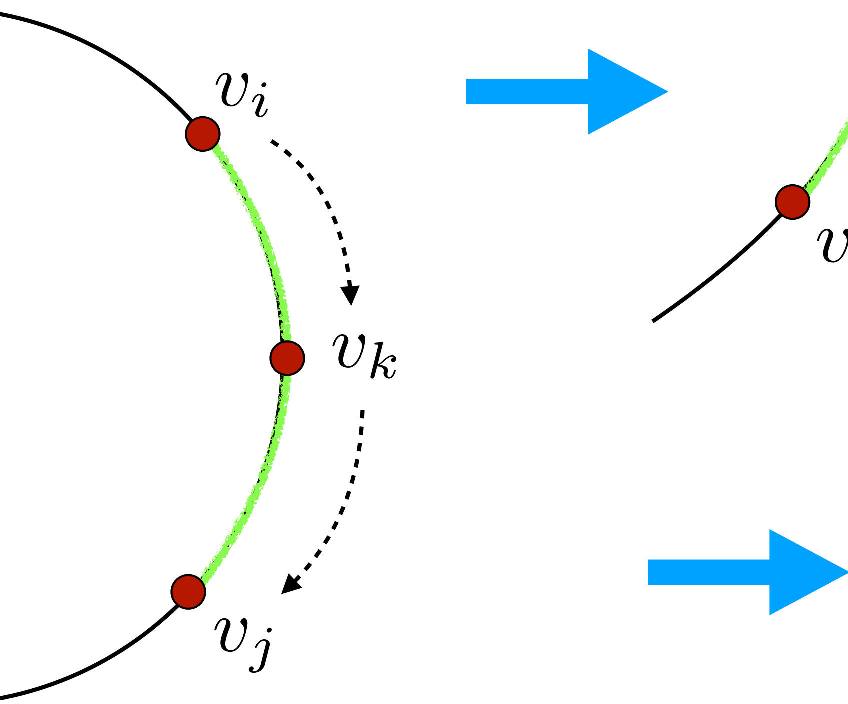
DP algorithm

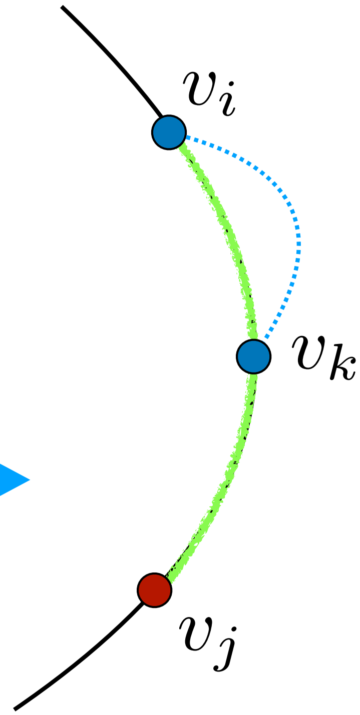


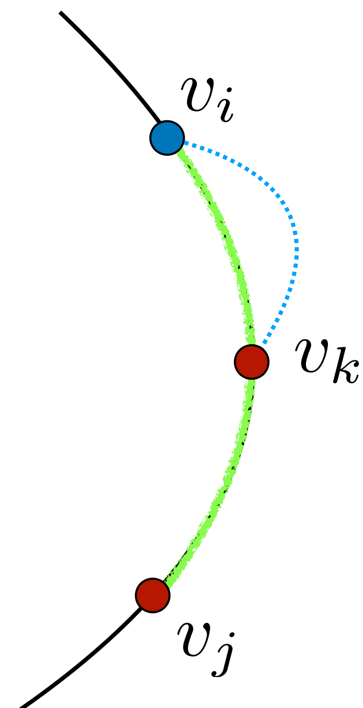


$$\mathcal{C}[v_i, v_j] = min_{\ i<k\leq j} \begin{cases} \mathcal{C}[v_{i+1}, v_j] & \text{if } v_i \ blue \\ \overline{\mathcal{C}}[\overline{v_i}, \overline{v_k}] + \mathcal{C}[v_k, v_j] + 1 & \text{if } v_k \ blue \\ \overline{\mathcal{C}}[\overline{v_i}, v_k] + \mathcal{C}[\overline{v_k}, v_j] + 1 & \text{if } v_k \ blue \\ \overline{\mathcal{C}}[\overline{v_i}, \overline{v_k}] + \mathcal{C}[\overline{v_k}, v_j] + 1 & \text{if } v_k \ red \\ \overline{\mathcal{C}}[\overline{v_i}, v_k] + \mathcal{C}[v_k, v_j] + 1 & \text{if } v_k \ red \end{cases}$$

DP algorithm







$$\mathcal{C}[v_i, v_j] = min_{\ i<k\leq j} \begin{cases} \mathcal{C}[v_{i+1}, v_j] & \text{if } v_i \ blue \\ \overline{\mathcal{C}}[\overline{v_i}, \overline{v_k}] + \mathcal{C}[v_k, v_j] + 1 & \text{if } v_k \ blue \\ \overline{\mathcal{C}}[\overline{v_i}, v_k] + \mathcal{C}[\overline{v_k}, v_j] + 1 & \text{if } v_k \ blue \\ \overline{\mathcal{C}}[\overline{v_i}, \overline{v_k}] + \mathcal{C}[\overline{v_k}, v_j] + 1 & \text{if } v_k \ red \\ \overline{\mathcal{C}}[\overline{v_i}, v_k] + \mathcal{C}[v_k, v_j] + 1 & \text{if } v_k \ red \end{cases}$$

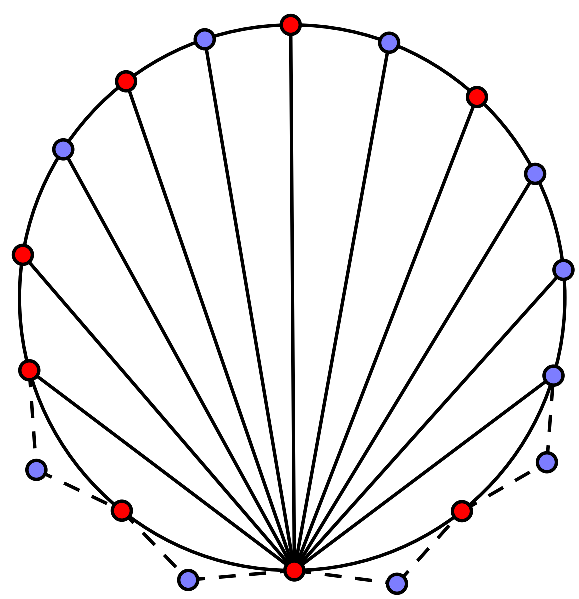
DP algorithm

Theorem 2.5. *Let $G = (V, E)$ be a MOP graph and C_G a set of parity constraints. Then, finding a compatible and disjoint graph $H = (V, E')$ with edge set E' of minimum size, such that $G' = G \cup H$ meets C_G, can be computed in $\mathcal{O}(n^3)$ time.*

Corollary 2.5.1. *Let $G = (V, E)$ be a MOP graph and C_G a set of parity constraints. Then, computing a topological plane matching M of $S(G, C_G)$ compatible and disjoint with G (if exists), can be done in $\mathcal{O}(n^3)$ time.*

Mobile-embedding augmentation of MOPs

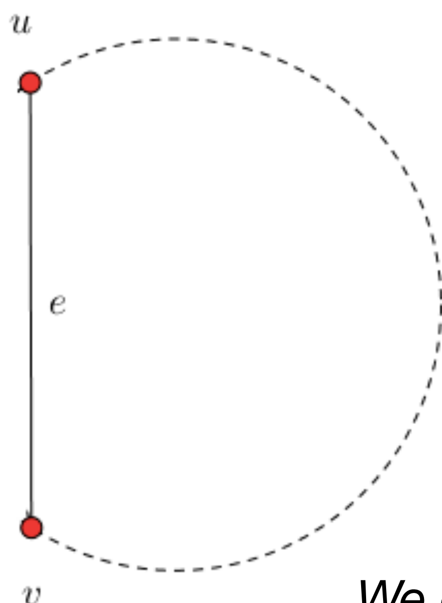
Non-augmentable graph family



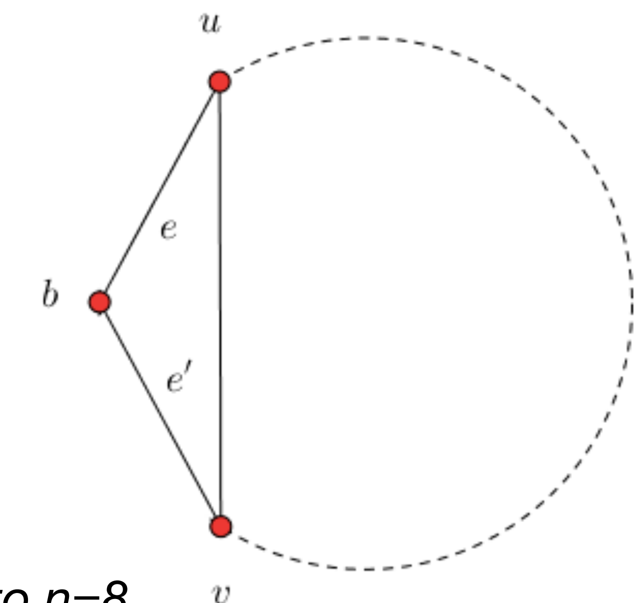
Start with possibly 4 blue ears

Mobile-embedding augmentation algorithm

$Suppose, \ \forall \, v \ \in \ V \ is \ red$



One isolated ear

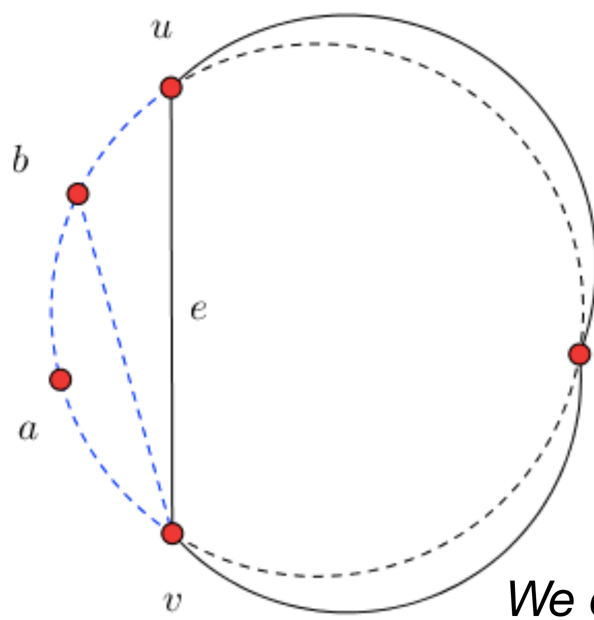


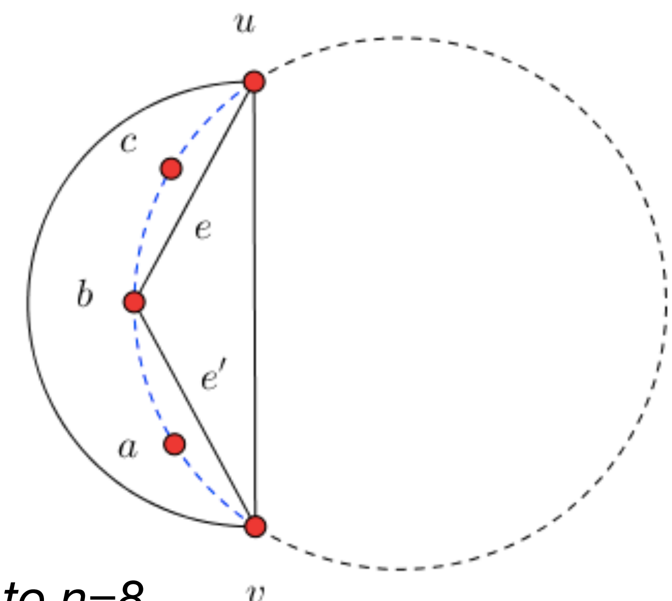
Two joint ears

We continue up to n=8

Mobile-embedding augmentation algorithm

$Suppose, \forall\, v\ \in\ V\ is\ red$





We continue up to n=8

One isolated ear

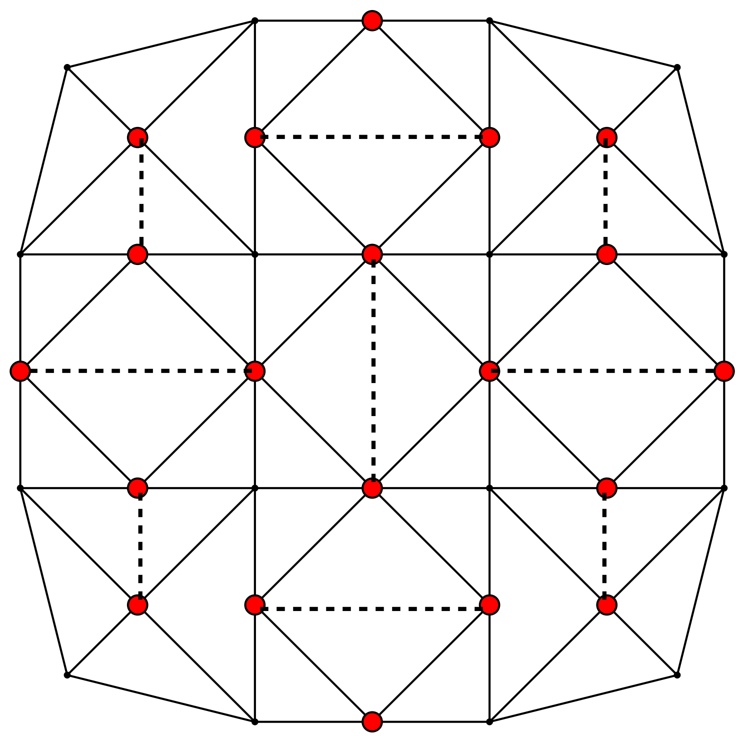
Two joint ears

Mobile-embedding augmentation algorithm

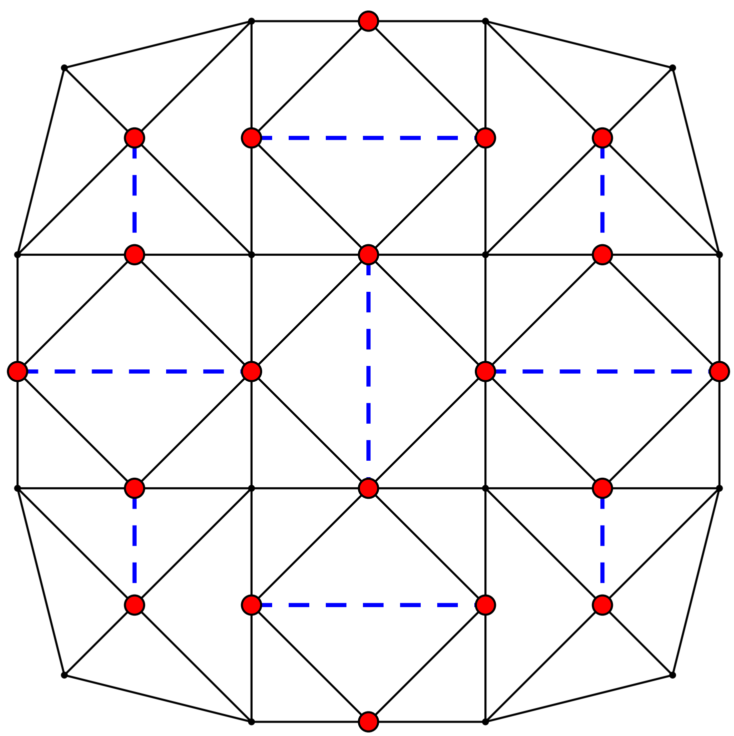
Theorem 3.1. *Let* $G = (V, E)$ *be a MOP graph and* C_G *a set of parity constraints, where* $S(G, C_G) = V$. *Then, deciding if* G *has an embedding such that there exists a compatible and disjoint topological graph* H, *such that* $G' = G \cup H$ *meets* C_G *can be computed in* $\mathcal{O}(n^2)$ *time.*

Complexity in geometric graphs

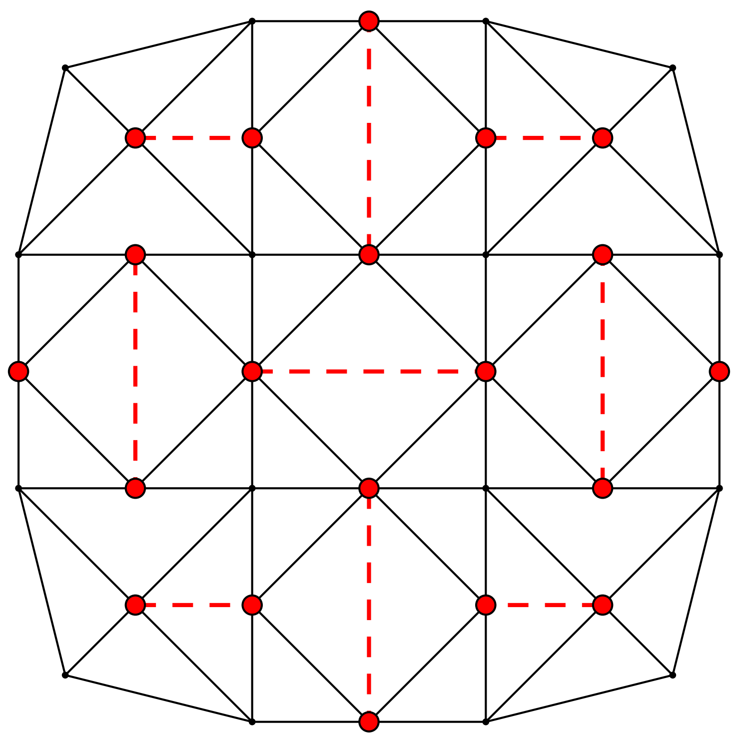
NP-Completeness of the augmentation decision problem



NP-Completeness of the augmentation decision problem

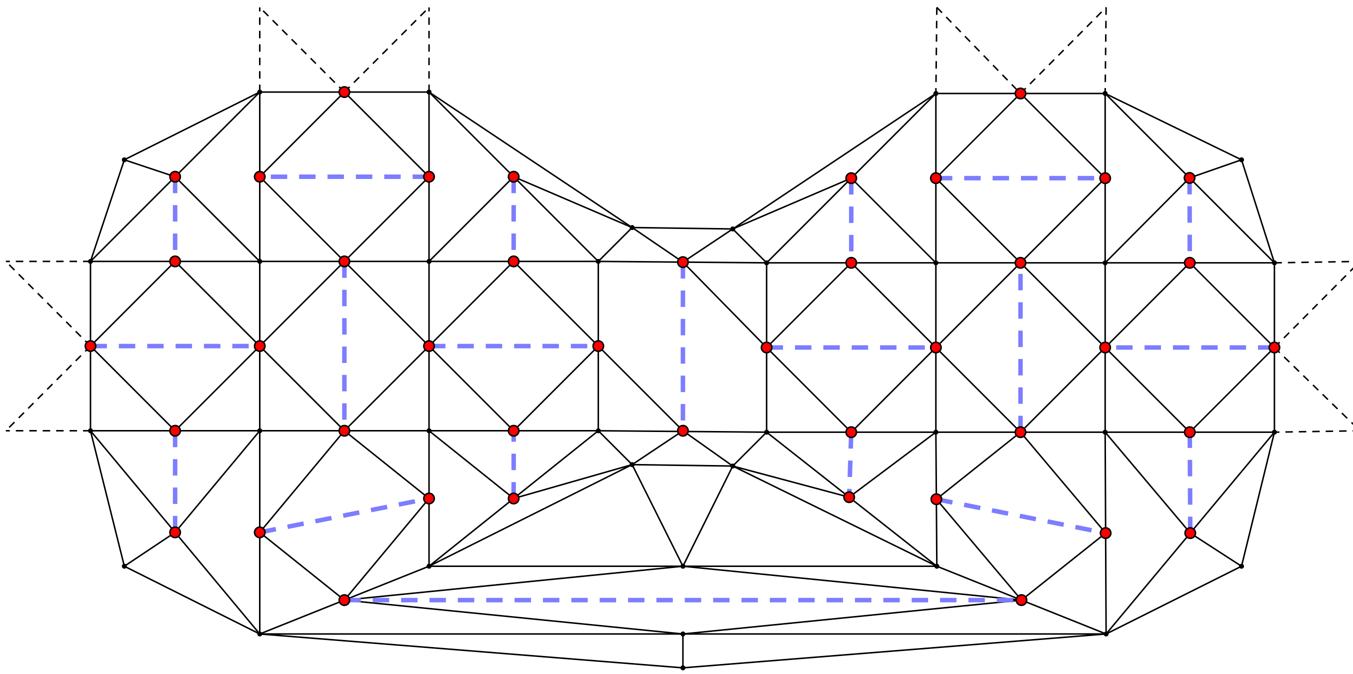


Negative augmentation



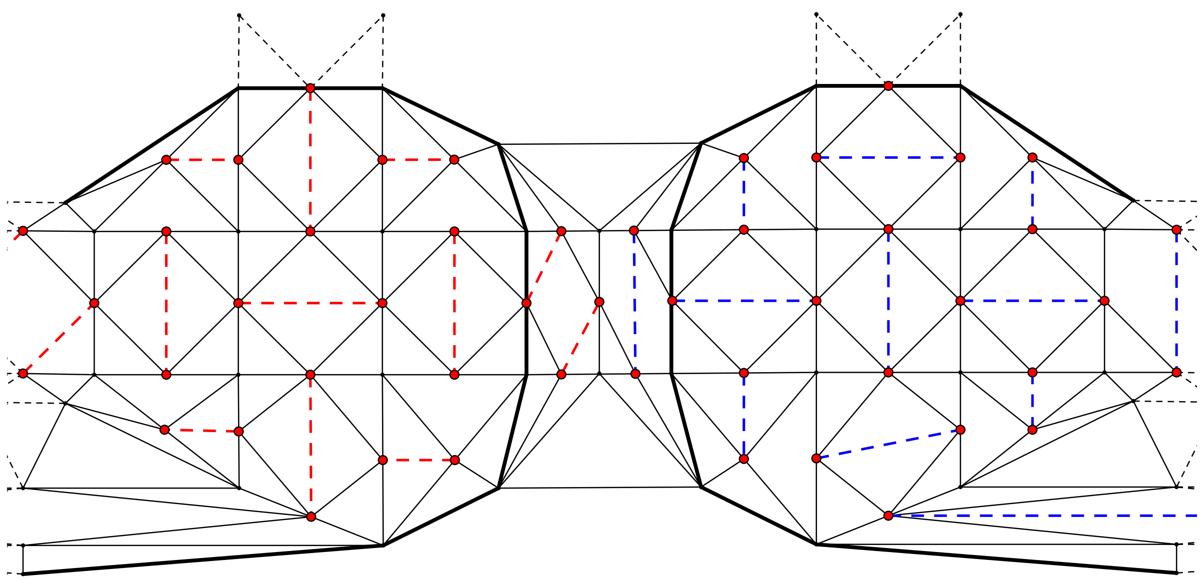
Positive augmentation

NP-Completeness of the augmentation decision problem



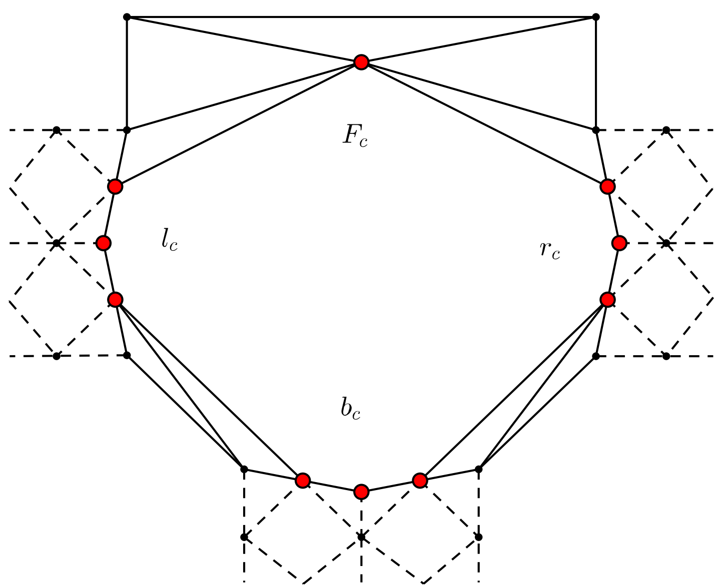
Literal gadget

NP-Completeness of the augmentation decision problem



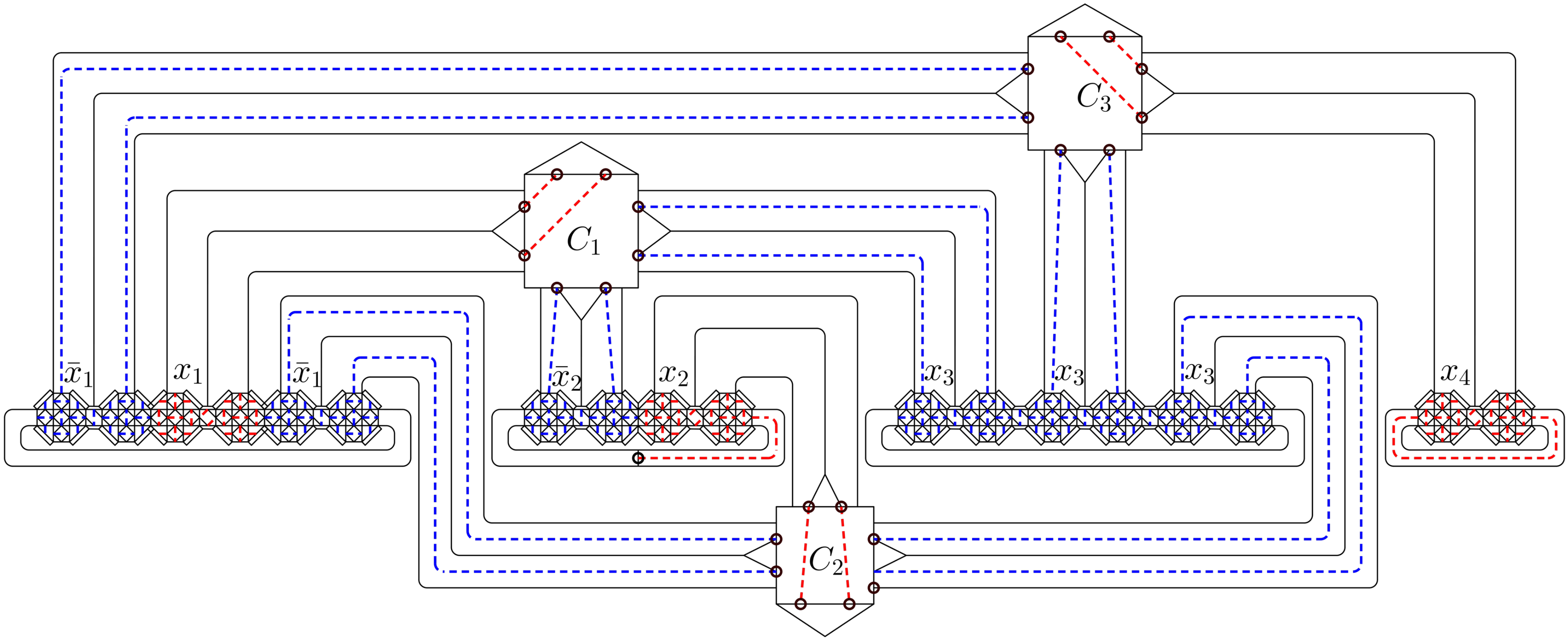
Connection of two literals

NP-Completeness of the augmentation decision problem



Clause gadget

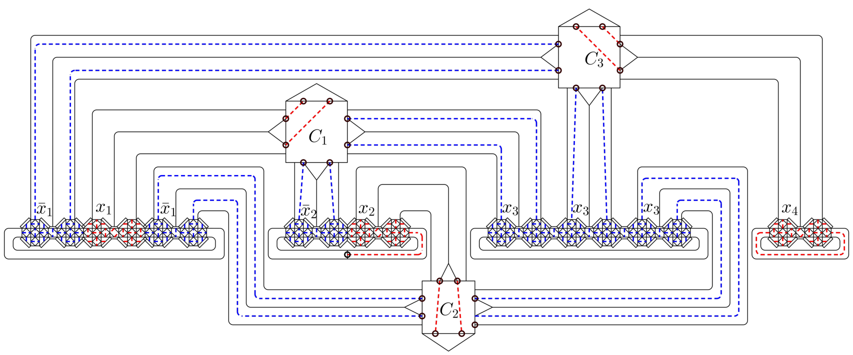
NP-Completeness of the augmentation decision problem



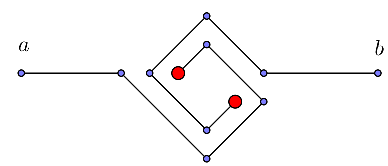
NP-Completeness of the augmentation decision problem

Theorem 4.1. *Let $G = (V, E)$ be a topological plane graph and C_G a set of parity constraints. Then, the problem of deciding if there exists a topological plane graph H disjoint and compatible with G, such that $G' = G \cup H$ meets C_G is $\mathcal{NP}$-Complete. The problem remains $\mathcal{NP}$-Complete even when $S(G, C_G) = V$.*

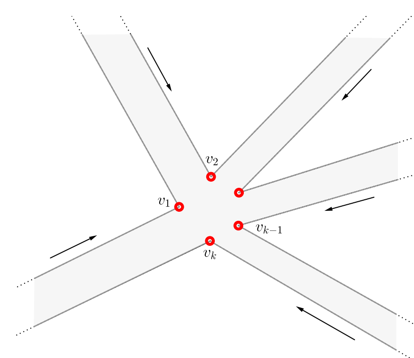
Hardness heritage of geometric trees & paths



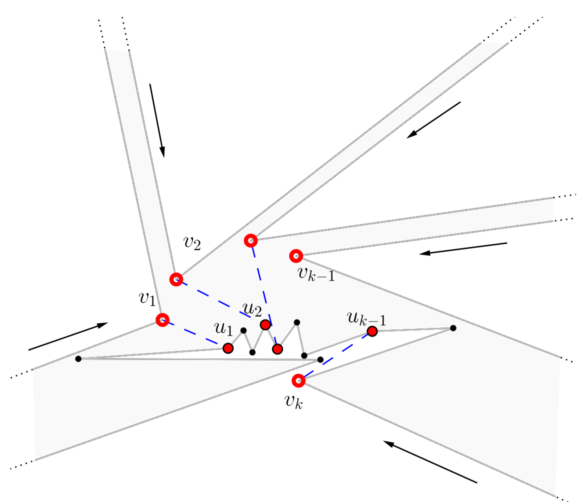
Remove cycles



Transform to a path



Modify the parity



Hardness heritage of geometric trees & paths

Theorem 4.2. *Let $T = (V, E)$ be a geometric plane tree. Then, the problem of deciding if T admits a compatible and disjoint perfect matching is $\mathcal{NP}$-Complete.*

Theorem 4.3. *Let $P = (V, E)$ be a geometric plane path. Then, the problem of deciding if P admits a compatible and disjoint perfect matching is $\mathcal{NP}$-Complete.*

Thanks!

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