# Optimal Coverage of a Tree with Multiple Robots

I. Aldana-Galván J.C. Catana-Salazar J.M. Díaz-Báñez
R. Fabila-Monroy M.A. Heredia F. Duque J. Urrutia
A. Ramírez-Vigueras

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# The problem

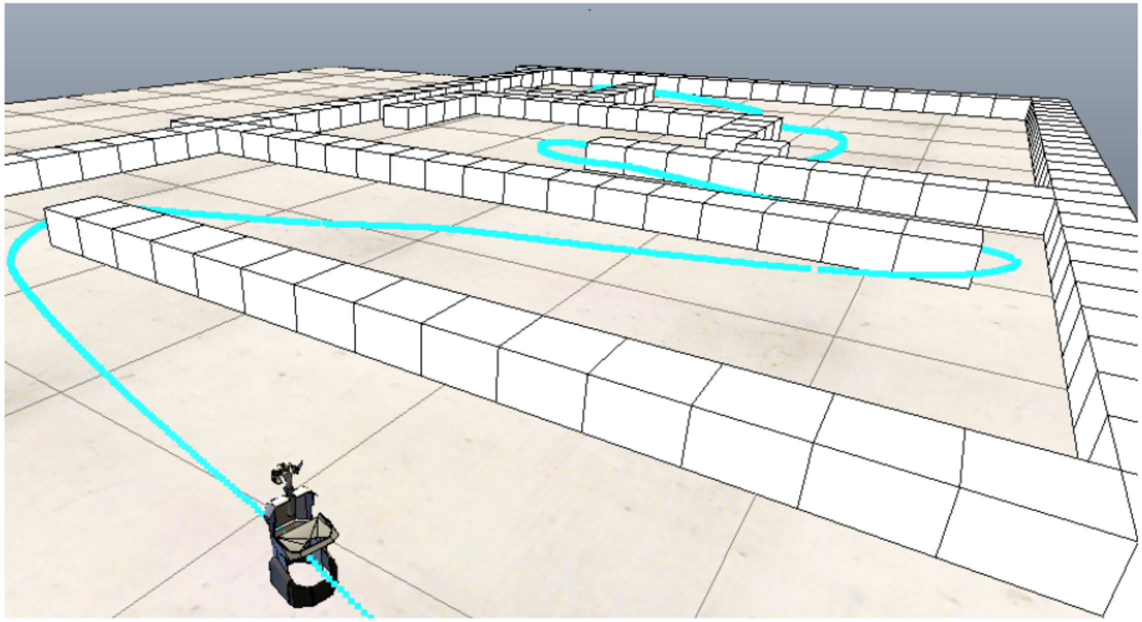
- Given a *terrain* and *k* robots, our task is to find the better way to cover the terrain.

- What the cost means?
    - Robot cost?
    - Minimum length walked.
    - Minimum time taken.

- Independent exploration.
- Rendezvous every *t* steps.
- Wireless communication.

# Motivation

- Plenty applications in robotics, path planning, motion planning, etc.

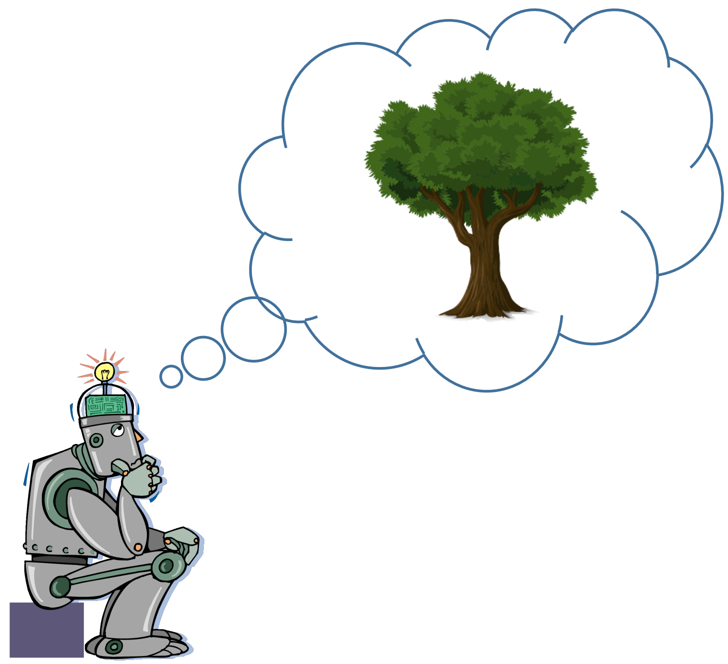


# Some considerations

- We’ll restrict to those terrains “looking” like a tree.
- Minimizing distance walked.
- Minimizing exploring time.
- Considering rendezvous at most every *t* steps.
- Each robot has a clock “synchronized”.
- Each edge has unit length distance

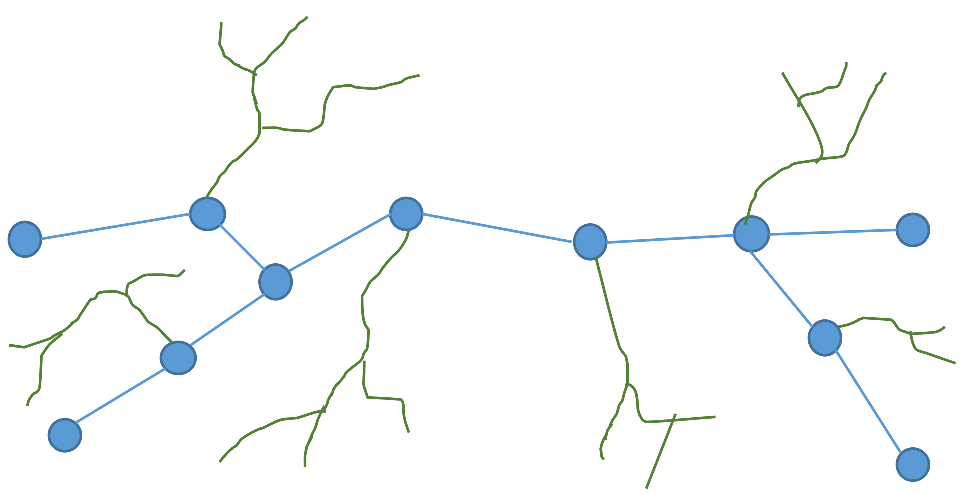
# Robot exploration minimizing distance walked

- ***PROBLEM 1***: Given a tree *T* and set of k robots, find the mínimum distance coverage of *T*.



# How does an optimal solution look like?

- Edges walked twice (**Forest**)
- Edges walked once per each robot (**Path**)



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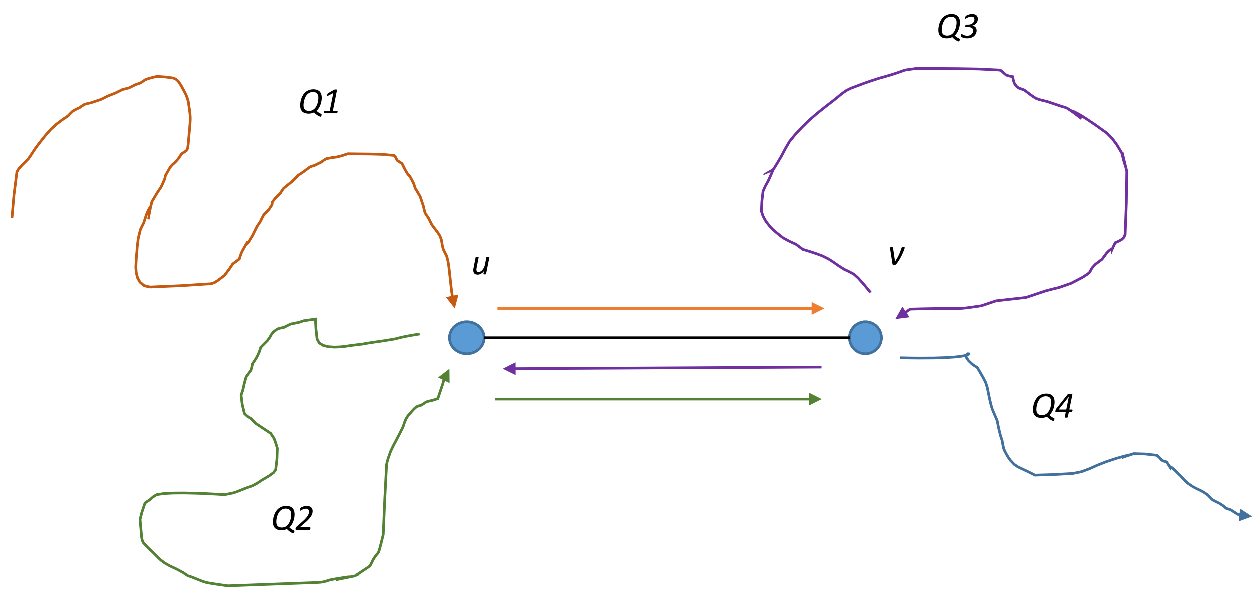
- Suppose an edge *uv* is walked by a robot at least three times:



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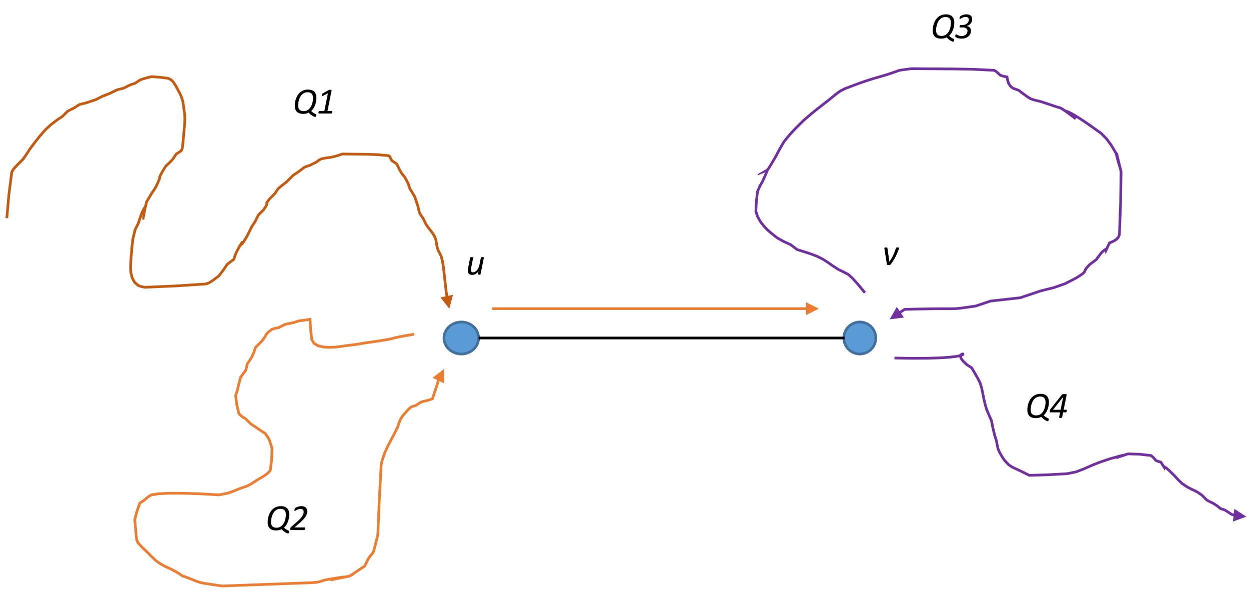

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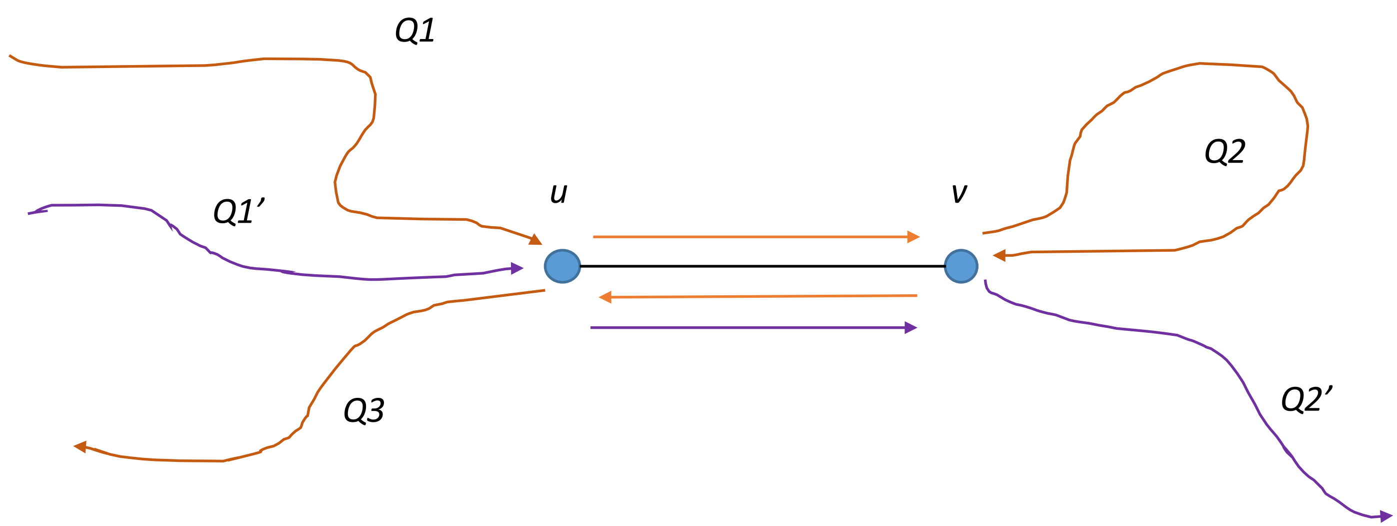

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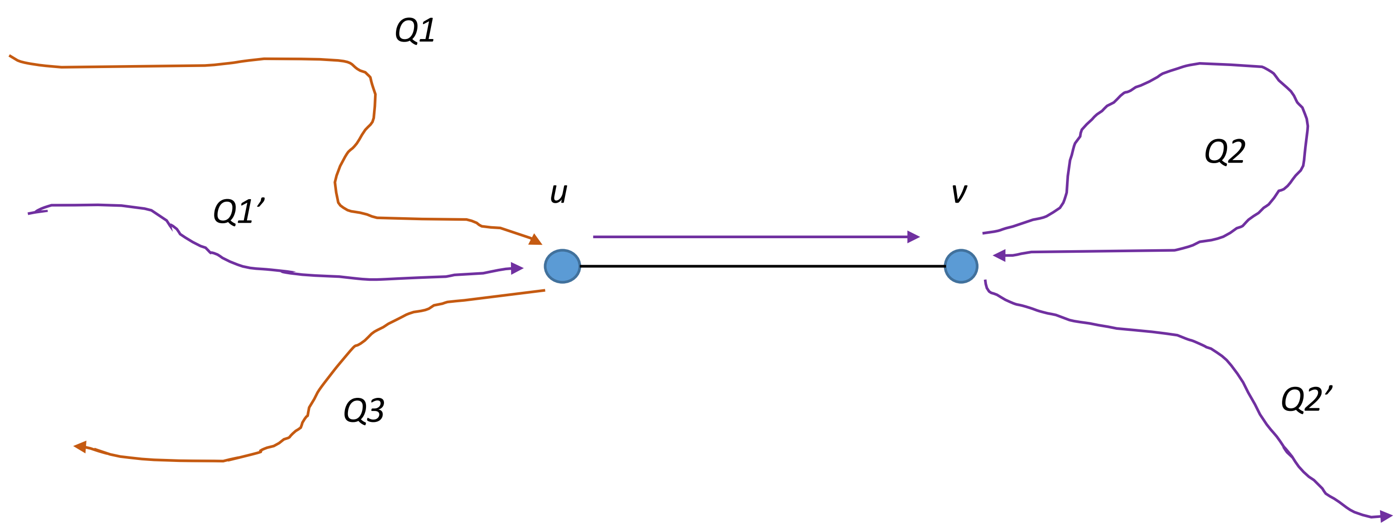

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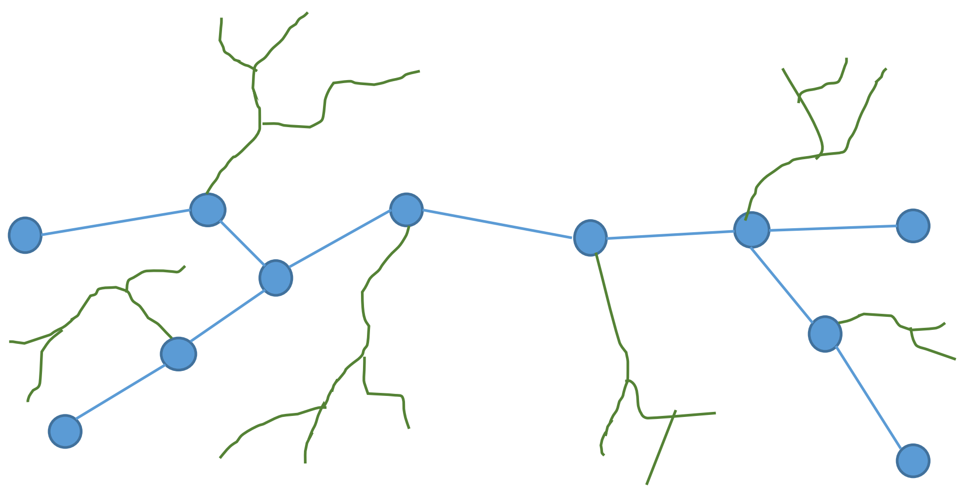
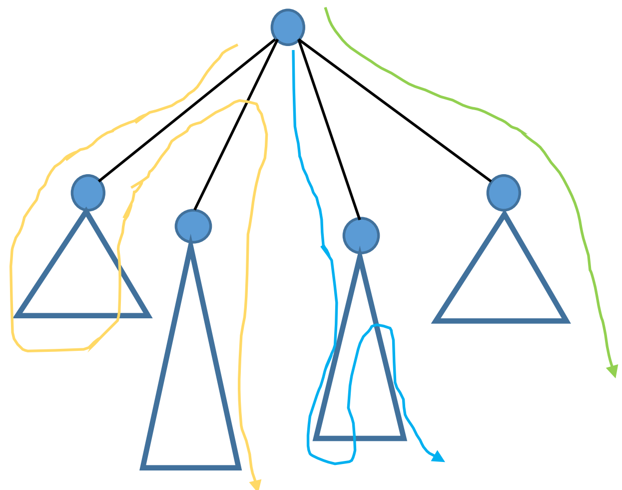


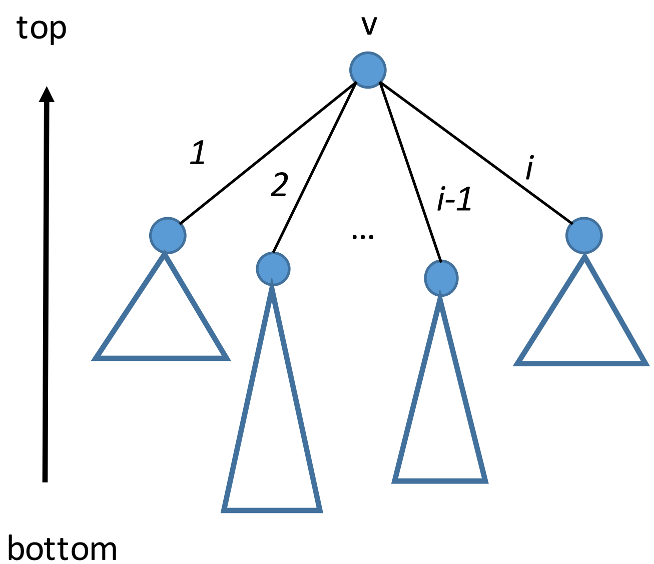
# All robots starting at a common vertex

Consider a vertex $u$ in $T$, and $k$ robots starting their exploration at $u$.



# All robots starting at a common vertex

$C[T_v[i], j] =$ Minimum cost of exploring $T_v$, considering until the $i$-th son, with $j$ robots.




$For\ each\ v$ in $V$:

$For\ each\ i$ in $deg(v)$:

$For\ each\ j \leq k$:

$For\ each\ s \leq j$:

$$C[T_v[i], j] = C[T_v[i-1], j-s] + C[T_{v_i}, s] + s$$

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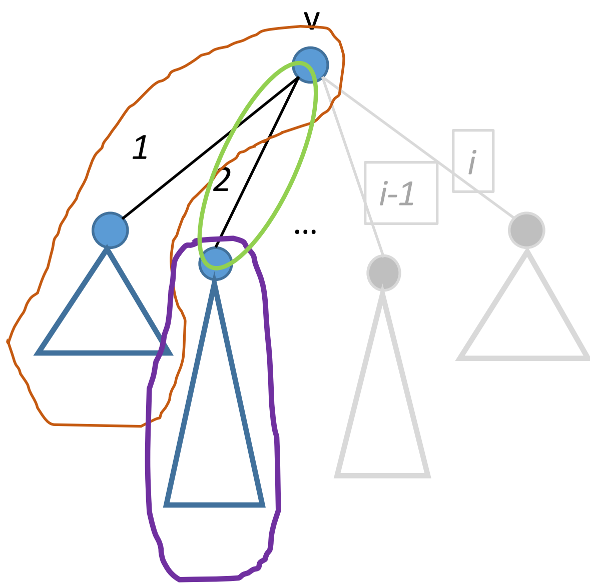


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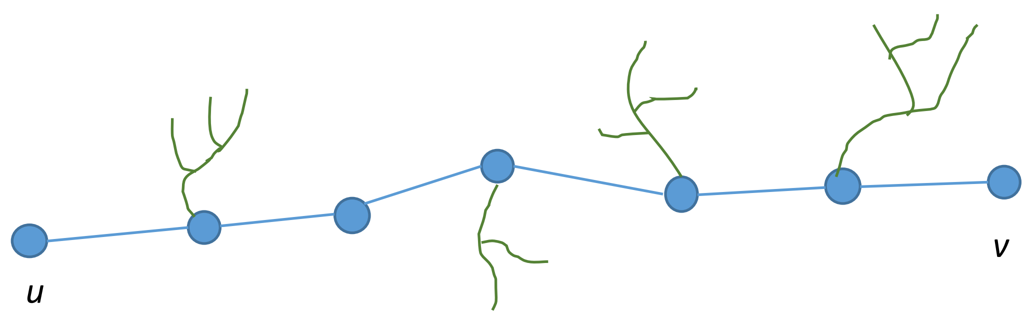
$$C[T_v[i], j] = C[T_v[i-1], j-s] + C[T_{v_i}, s] + s$$

# All robots starting at a common vertex

**Theorem** *Let $k$ be a positive integer, and let $u$ be a vertex of $T$. Then for every $1 \leq i \leq k$, a minimum length covering strategy for $T$ with $i$ robots, all starting at $u$, can be computed $O(k^2 n)$ total time.*

# All robots starting at a common vertex 2

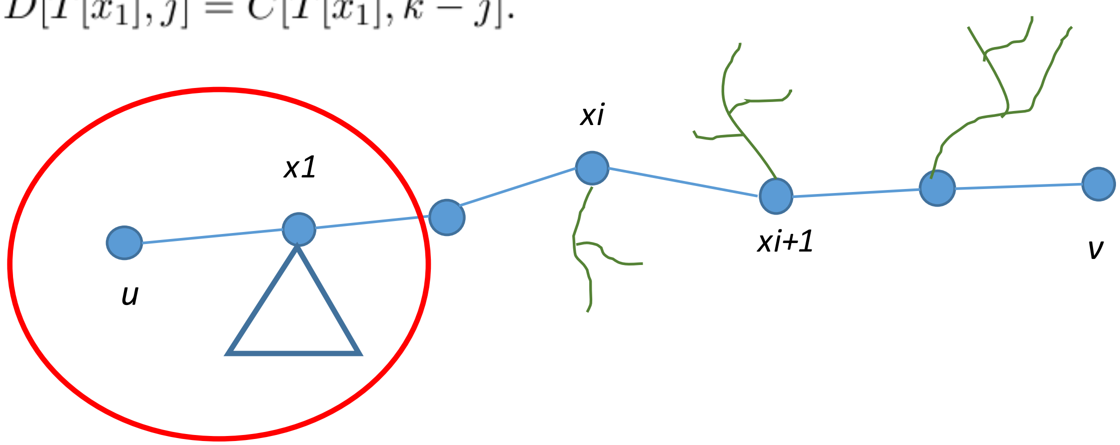
Consider two vertices $u,v$ in $T$, $k$ robots starting their exploration at $u$, and $j$ of them finishing at $v$.



# All robots starting at a common vertex 2

$D[T[x_i], j]$ = Minimum cost of exploring $T[x_i]$ with $k$ robots, such that $j$ finish at $v$.

$$D[T[x_1], j] = C[T[x_1], k - j].$$



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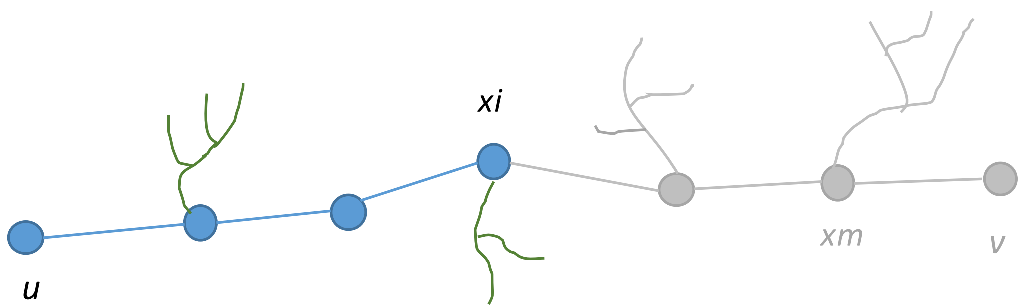


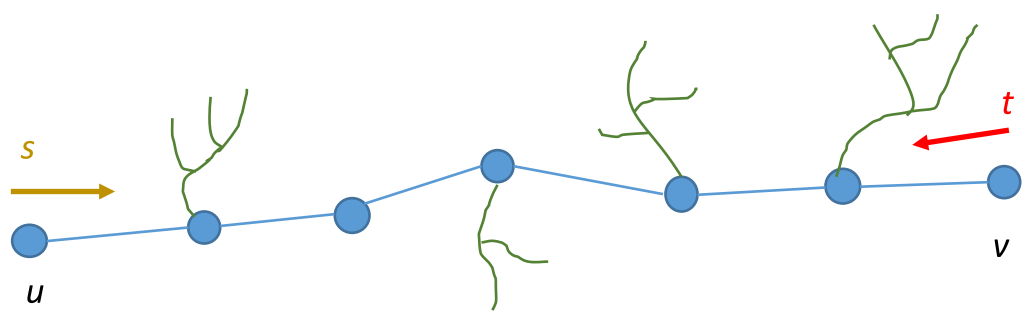

*For each* $i \leq m$:
  *For each* $j \leq k$:
    *For each* $s \leq j$:

$$D[i, j] = D[i-1, s] + s + C[i, s-j].$$

$$O(k^2 n)$$

# Robots starting at a two vertices

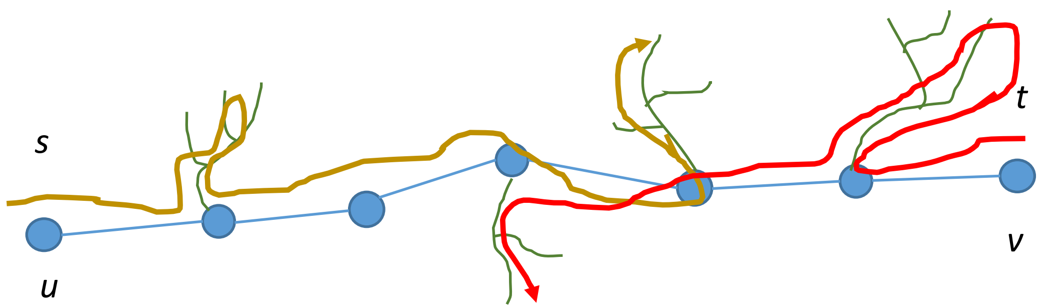
Consider two vertices *u,v* in *T*, *s* robots starting at *u* and *t* of them at *v*.



# Robots starting at a two vertices

Consider two vertices *u,v* in *T*, *s* robots starting at *u* and *t* of them at *v*.

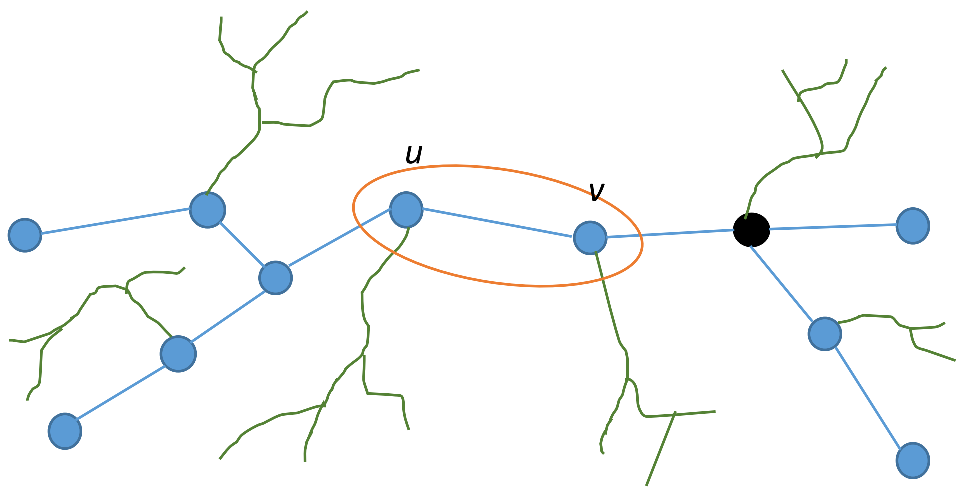
$C[T_v[i], j]$ $D[T[x_i], j]$




$O(k^2 n)$

# Robots starting at k vertices.

- Let *H* be the minimum graph connecting the *k* vértices.



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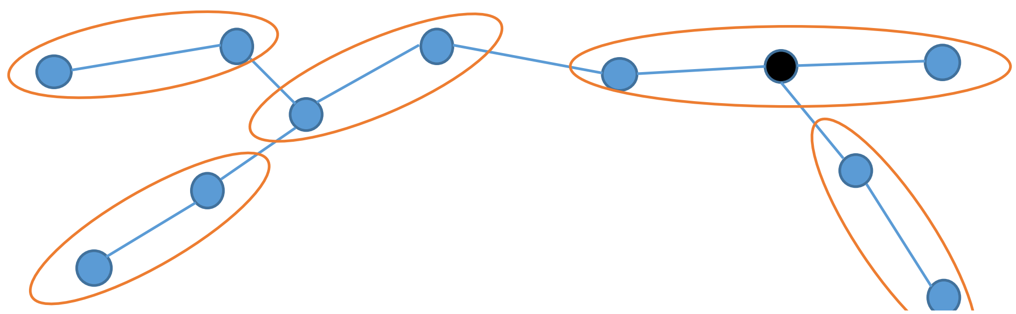
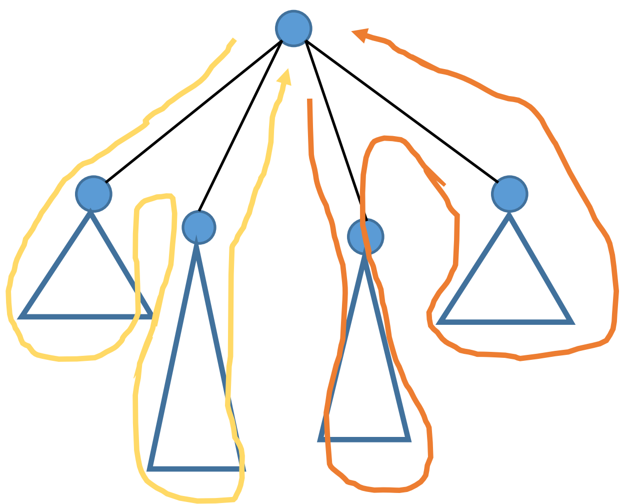


**Theorem** *A minimum length covering strategy for $T$, with $k$ robots, with given starting positions, can be computed $O(k^3 n + 2^k k^{k+1})$ time.*

# Robot exploration minimizing distance walked with rendezvous

- ***PROBLEM 2***: Given a tree $T$ and set of $k$ robots, find the minimum distance coverage of $T$, when the robots rendezvous at most every $t$ steps and they start at a vertex $v$.



# Robot exploration minimizing distance walked with rendezvous

- Given a tree *T* and set of *k* robots, find the minimum distance coverage of *T*, when the robots rendezvous at most every *t* steps and they start at a vertex *v*.

**Problem (3-PARTITION)** *Let $B$ be a positive integer. Let $A$ be a set of $3m$ positive integers, such that $\frac{B}{4} < a < \frac{B}{2}$ for all $a \in A$, and $\sum_{a \in A} a = mB$. The* **3-PARTITION** *problem asks if $A$ can be partitioned into $m$ sets $A_1, \ldots, A_m$ such that for each $1 \leq i \leq m$, $\sum_{a \in A_i} a = B$.*

# Robot exploration minimizing distance walked with rendezvous

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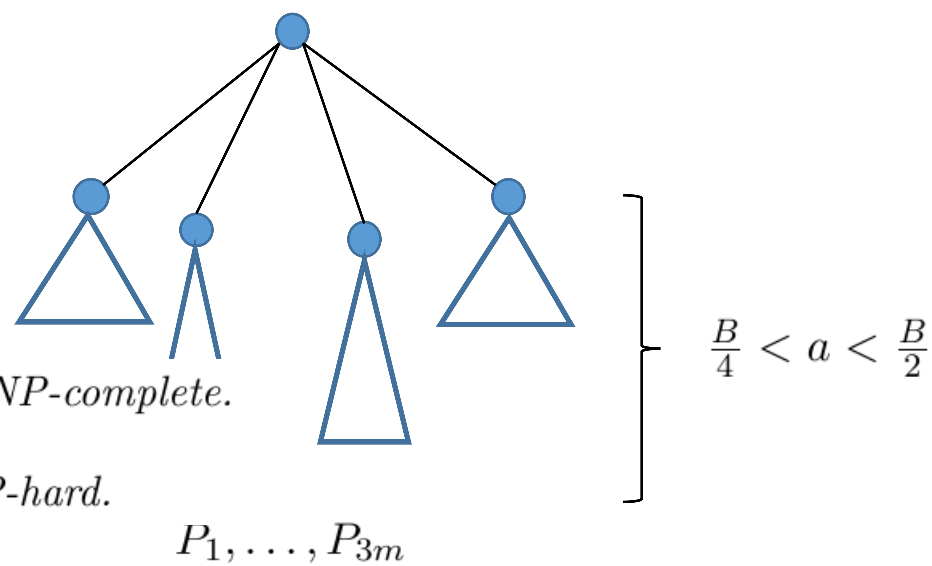
$$A := \{a_1, \ldots, a_{3m}\}$$




**Theorem** *The MLCP-R Problem is NP-complete.*

**Theorem** *The MTCP Problem is NP-hard.*

*Thanks!*